FRACTALS-3

**Methods of calculating fractal dimension**

**Generalized dimension**

In computing the box-counting dimension, the number of points inside box is not considered and all boxes (with few and with many points) have the same weight in calculation of fractal dimension [1].

The generalized dimension [2,3] does take into account the number of points in the box.

* Count the number of points (belonging to the structure)  in th box.
* Probability of a randomly chosen point on the structure to be in th box is  whereis the total number of points on the structure.
* The generalized dimension is defined as

.

For uniform fractals (monofractals) with all  equal, does not vary with .

For nonuniform fractals variation of  with quantifies the nonuniformity, decreases with increasing .

The parameter  provides a microscope for exploring different regions of the singular measure; positive ’s accentuate the denser regions and negative ’s accentuate the rarer ones.

The maximum dimension  is associated with the least-dense points on the fractal and the minimum dimension corresponds to the most-dense points.

The structures that are characterized by the full spectrum of generalized dimensions from  to are called multifractals.

Forgeneralized dimension corresponds to the capacity (box-counting) dimension .

For generalized dimension corresponds to the information dimension .

Forgeneralized dimension corresponds to the correlation dimension .



**Holder exponent and Singularity spectrum**

Halsey et al. [4] introduced a change of variables that provides a new interpretation of generalized dimension.

Let  and take Legendre transformations from the variables into a new set of variables 

,  and , 

where is a scaling index that describes local scaling in *i*th box : .

The number of boxes with a same scaling index is .

is the Hausdorff dimension of the set of points with local scaling index .

The exponent  (also called singularity strength or local Holder exponent) takes values from the interval.

The function (also called singularity spectrum or multifractal spectrum) is usually a single humped function with maximum at 

For monofractals, spectrum is reduced to the single point.

For monofractals is a linear function, for multifractals is a convex function.

Monofractals: =const, is a linear function, single point

Multifractals: is a decreasing function, is a convex function, is a single humped function

[1] J. Theiller, Estimating fractal dimension, J. Opt. Soc. Am. 7, 1055-1073, 1990.

[2] H.G.E. Hentschel, I. Procaccia, The infinite number of generalized dimensions of fractals and strange attractors, Physica D 8, 435-444, 1983.

[3] P. Grassberger, Generalized dimensions of strange attractors, Physics Letters A 97, 227-230,1983

[4] T. C. Halsey, M. H. Jensen, L. P. Kadanoff, I. Procaccia, B.I. Shraiman, Fractal measures and their singularities: the characterization of strange sets, Physical Review A 33, 1141-1151, 1986.